UC Berkeley EECS
Lecturer
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Computational Structures in Data Science

Recursion II

Announcements

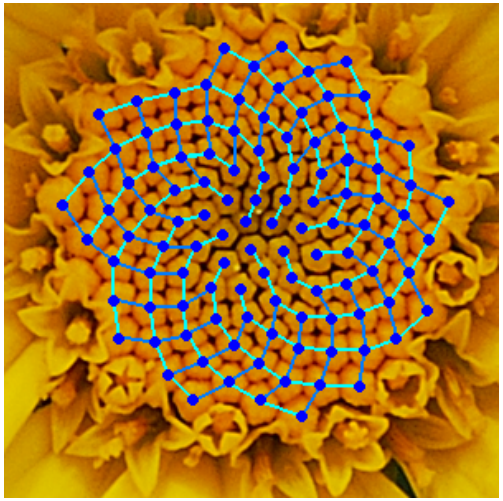
- Practice Midterm is worth extra credit!
  - Practice with the exam tool (exam.cs61a.org)
  - Practice with creating a Zoom recording
  - You will submit the exam online, then submit a link to a recording.
  - Will be worth 2 of 56 points.
  - Should only take you 10 minutes. ☺
- Recursion is on the midterm, but not super advanced recursion.

Learning Objectives

- Write a recursive function
- Understand the base case and a recursive case
- Apply Recursive Functions to lists

Why Recursion?

- Recursive structures exist (sometimes hidden) in nature and therefore in data!
- It's mentally and sometimes computationally more efficient to process recursive structures using recursion.
- Sometimes, the recursive definition is easier to understand or write, even if it is computationally slower.



The Recursive Process

- Recursive solutions involve two major parts:
  - Base case(s), the problem is simple enough to be solved directly
  - Recursive case(s). A recursive case has three components:
    - Divide the problem into one or more simpler or smaller parts
    - Invoke the function (recursively) on each part, and
    - Combine the solutions of the parts into a solution for the problem.

In words

- The sum of no numbers is zero
- The sum of 1^2 through n^2 is the
  - sum of 1^2 through $(n-1)^2$
  - plus n^2

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually stops on the simple base case

Recursion With Lists

```
def first(s):
    """Return the first element in a sequence."""
    return s[0]
def rest(s):
    """Return all elements in a sequence after the first"""
    return s[1:]

def min_r(s):
    “”””Return minimum value in a sequence.”””
    if
    else:
```

Base Case

Recursive Case

- Recursion over sequence length, rather than number magnitude

Recursion With Strings

```
def reverse(s):
    """
    >>> reverse('hello')
    'olleh'
    >>> reverse(reverse('hello'))
    'hello'
    """
    if not s:
        return ''
    return
```

Recursive Case

- Recursion over sequence length, rather than number magnitude

Local variables

- Each call has its own "frame" of local variables
- Let's see the environment diagrams

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

https://goo.gl/CiFaUJ

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

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pythontutor.com

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

Edit code

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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that has just executed
: line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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that has just executed
t line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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that has just executed
t line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1

Environments Example

Python 3.3

```
1 def sum_of_squares(n):
2     n_squared = n**2
3     if n == 1:
4         return 1
5     else:
6         return n_squared + sum_of_squares(n-1)
7
8 sum_of_squares(3)
```

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e that has just executed
xt line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4

f3: sum_of_squares [parent=Global]
n 1
n_squared 1
Return value 1

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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that has just executed
ct line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n 3
n_squared 9

f2: sum_of_squares [parent=Global]
n 2
n_squared 4
Return value 5

f3: sum_of_squares [parent=Global]
n 1
n_squared 1
Return value 1

Environments Example

Python 3.3

```
1  def sum_of_squares(n):
2      n_squared = n**2
3      if n == 1:
4          return 1
5      else:
6          return n_squared + sum_of_squares(n-1)
7
8  sum_of_squares(3)
```

Edit code

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e that has just executed
xt line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]

n	3
n_squared	9
Return value	14

f2: sum_of_squares [parent=Global]

n	2
n_squared	4
Return value	5

f3: sum_of_squares [parent=Global]

n	1
n_squared	1
Return value	1

Extra Examples (We'll cover later, too.)

Tree Recursion: Preview for Post-Midterm

- Break the problem into multiple smaller sub-problems, and Solve them recursively

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def quicksort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = s[0]
        smaller, bigger = split(pivot, s[1:])
        return quicksort(smaller) + [pivot] + quicksort(bigger)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

QuickSort Example