Computational Structures in Data Science

Tree
Recursion

Learning Objectives

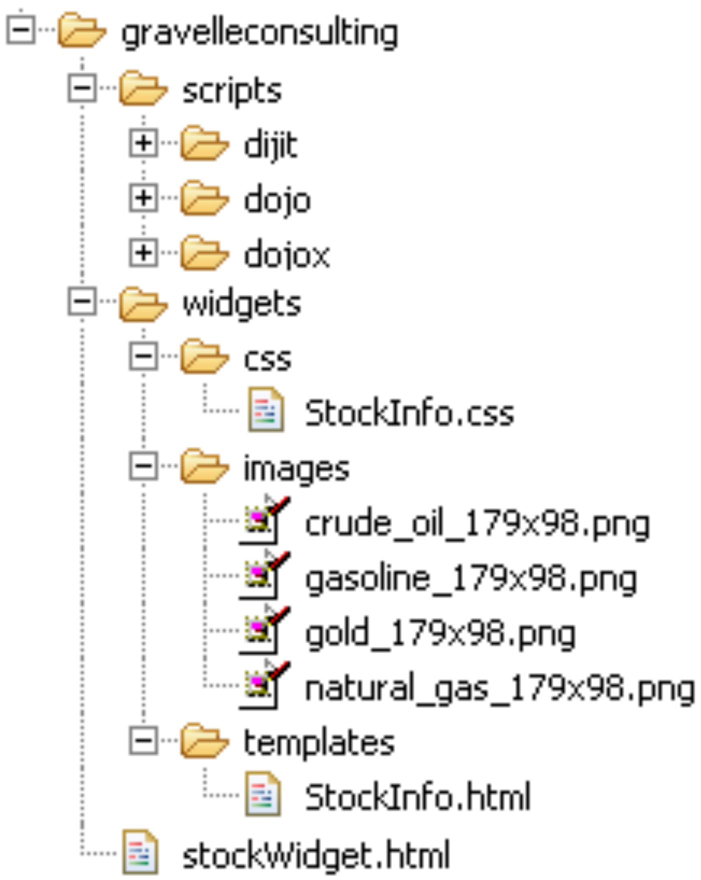
- Write Recursive functions with multiple recursive calls
- Understand Recursive Fibonacci
- Understand the the `count_change` algorithm
- Bonus: Use multiple recursive calls in to sort a list.

Tree Recursion

- Tree Recursion in which involves multiple recursive calls to solve a problem.
- Drawing out a function usually looks like an “inverted” tree.
- Revisit the "vee" program from lecture 11.
- Many of these programs *can't* be written with iteration very easily
- You *can* solve any problem with recursion or iteration, but this technique that makes some problems simpler.

Example I

List all items on your hard disk



- Files
- Folders contain
  - Files
  - Folders

```
def process_directory(directory):
    for item in directory:
        if is_file(item):
            process_file(item)
        else:
            process_directory(item)
```

Demo – "Walking" a Directory

- This example relies on OS-level code, which we don't code.
- But the recursive template is quite simple.
- `>>> walk_directory('/Users/Michael/Desktop/Photography', max_depth = 3)`

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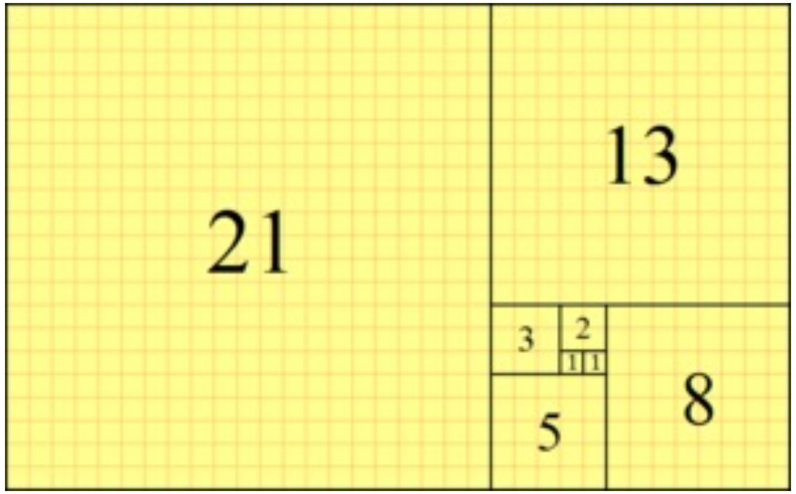
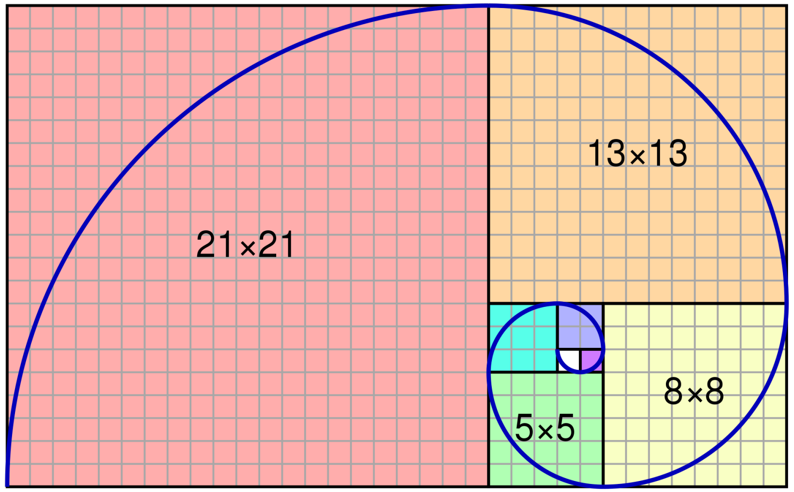
The Fibonacci Sequence

The Fibonacci Sequence

- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89…

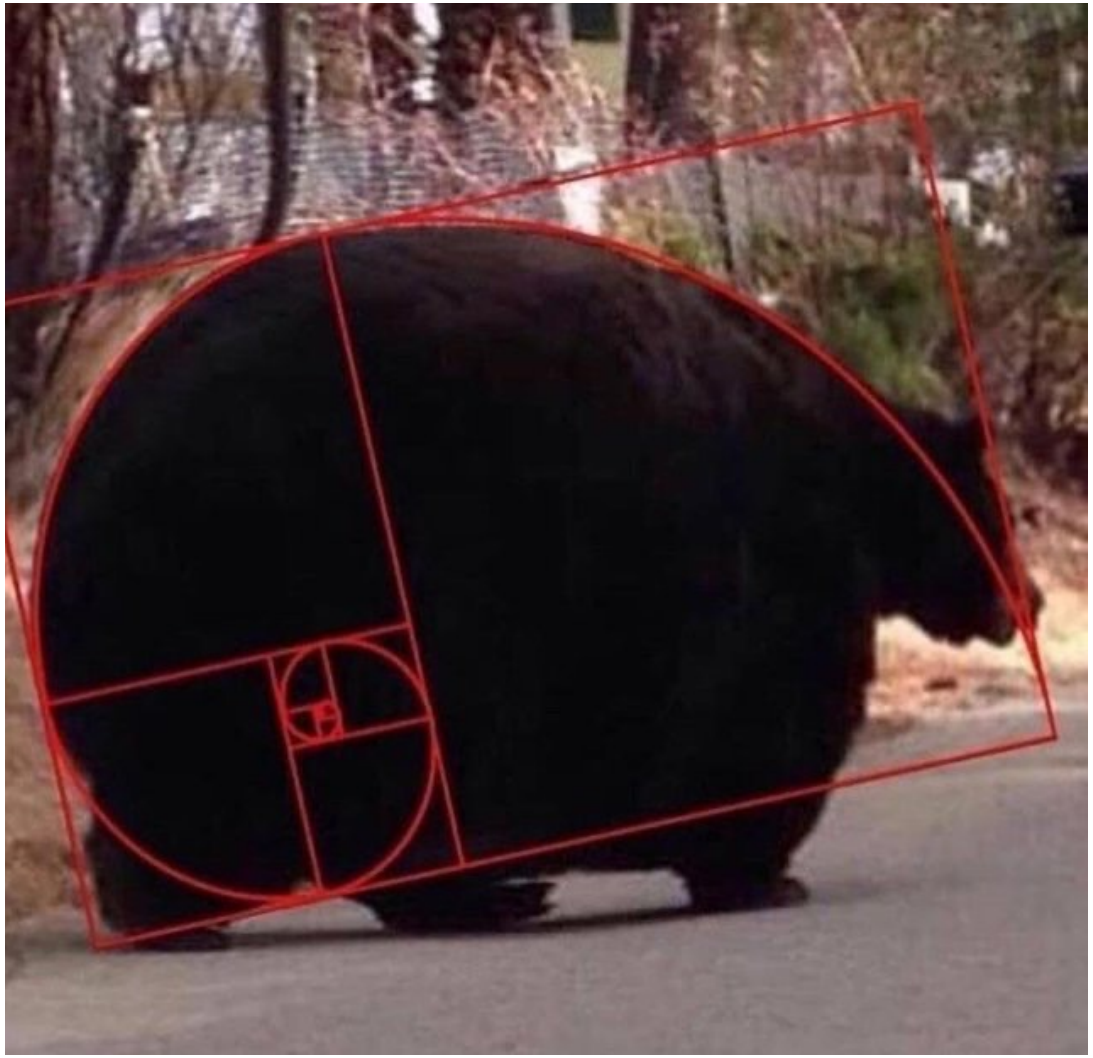
- F0 = 0, F1 = 1
- Fn = F(n-1) + F(n-2)





Golden Spirals Occur in Nature

GO BEARS



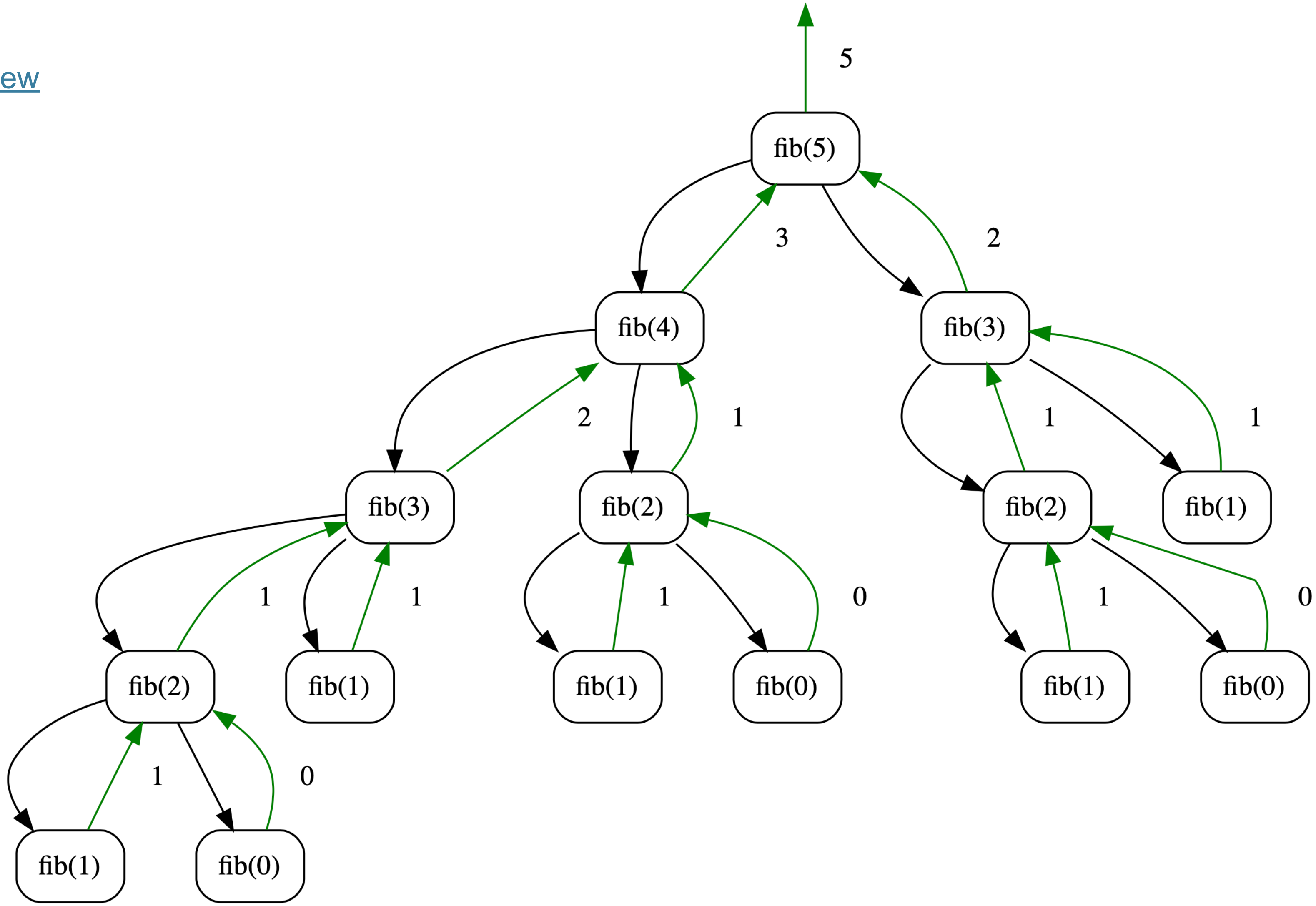
Fibonacci Code

```
fibonacci(n) = fibonacci(n-1) + fibonacci(n-2)
      where fibonacci(1) == 1 and fibonacci(0) == 0
```

```
def fib(n):
    """
    >>> fib(5)
    5
    """
    if n < 2:
        return n
    return fib(n - 1) + fib(n - 2)
```

Visualizing Fib Recursion:

Interactive View



But what about the iterative version?

- In practice, recursive fib is *slow!*
- We can write the program using a for loop.
- How do we translate this? You've done it before!
  - Technique is called "dynamic programming".

```
def iter_fib(n):
    (n_1, n_2) = (0, 1)
    for i in range(0, n):
        # This computes n_1+n_2 before updating n_1
        (n_1, n_2) = (n_2, n_1 + n_2)
    return n_1
```

What's Similar to Fibonacci?

- Many number sequences have similar properties
  - Catalan numbers
  - Pascal's Triangle
- "Branching" Patterns in Biology:
  - (Real) Tree branches
  - Veins in leaves
  - Romanesco Broccoli
  - Population growth of animals over N generations
  - Some of these structures can be modeled recursively

Computational Structures in Data Science

Count Change

Counting Change

- **Problem Statement:**
- Given (an infinite number of) coins, (25¢, 10¢, etc) how many different ways can I represent 10¢?
  - e.g. 5¢ can be made 2 ways: 1 nickel, or 5 pennies
  - 10¢ can be made 4 ways: [1x 10¢, 2x 5¢, 1 5¢ + 5 1¢, 10x 1¢]
  - Order doesn't matter, 5¢ + 5 1¢ is the same as 5 1¢ + 5¢
- **How do we solve this?**

Why use problems like count change?

- We're partitioning coins, but these could be bills, or other currency
- Explore of problem like [count_partitions]
- Many tree recursive questions follow a similar *recursive* step
  - Notice how instead of a conditional, we combine the results of two possible options
  - We make recursive calls for all possible outcomes, then the *base case(s)* handle the conditional logic.

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Counting Change

- change for 25¢ using [25, 10, 5] → 4

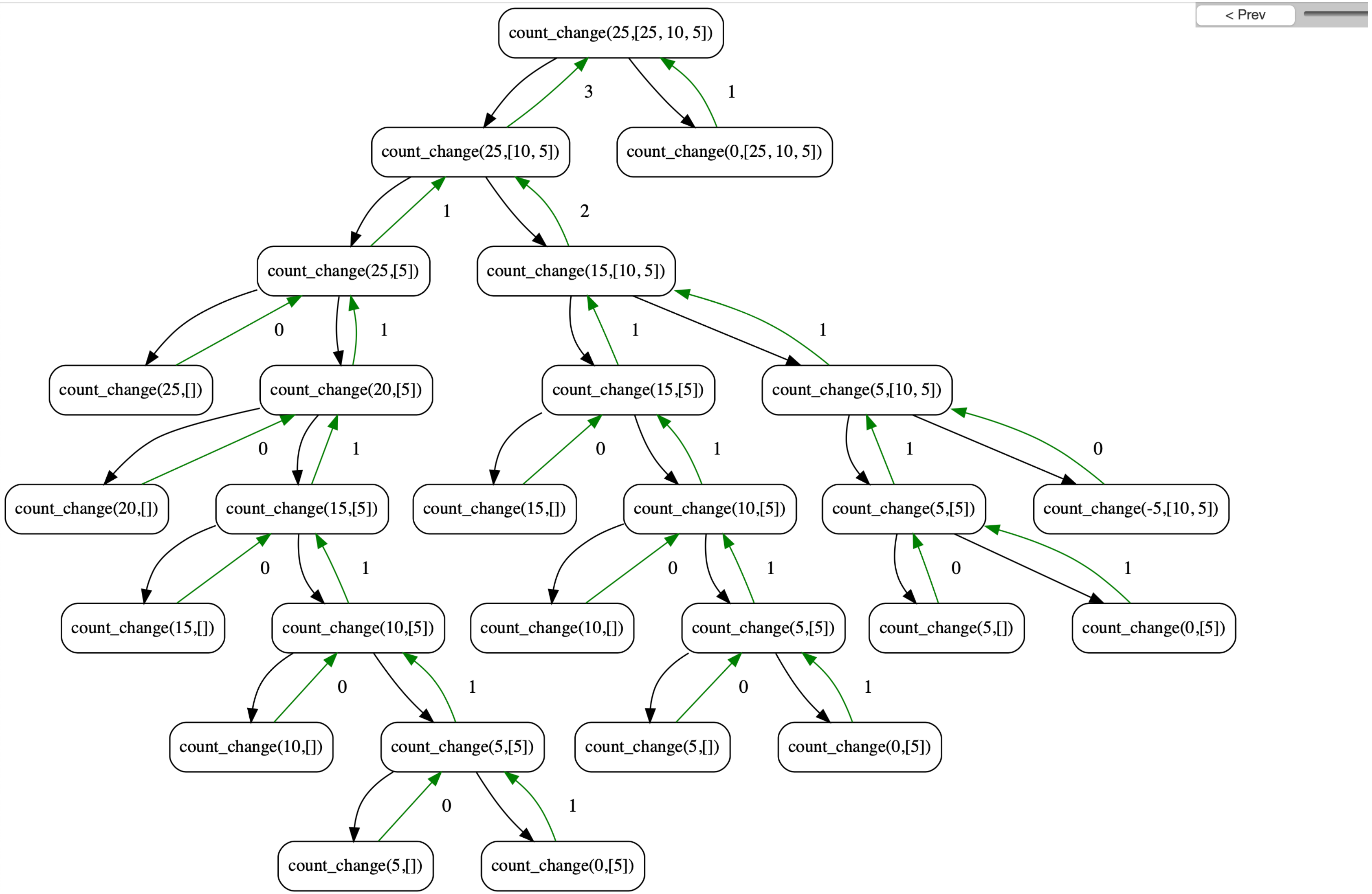
- What do we return?
  - 1 if valid count
  - 0 otherwise
- What are possible “smaller” problems?
  - Smaller amount of money → use coin
  - Fewer coins → “discard” coin
- What is our base case?
  - valid count: value is 0
  - invalid count: value is < 0, or no coins left
- **Recursion:**
  - <u>Divide</u>: split into two problems (smaller amount & fewer coins)
  - <u>Combine</u>: addition (# of ways)

count_change code

```
def count_change(value, coins):
    """
    >>> denominations = [50, 25, 10, 5, 1]
    >>> count_change(7, denominations)
    2
    """
    if value < 0 or len(coins) == 0:
        return 0
    elif value == 0:
        return 1
    using_coin = count_change(value - coins[0], coins)
    not_using_coin = count_change(value, coins[1:])
    return using_coin + not_using_coin
```

Visualizing Count Change

- [Interactive view]()



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There are many more recursive problems!

- The Knapsack Problem: Maximize the value of items thrown in a bag up to some "weight"
- Anything relating to Family Trees, Relationships, Social Networks
  - Count the Nth degree of "followers of followers" of some one
  - Many of these involve *graphs* which you'll learn in CS61B
- Subsets, Combinations, Permutations
- Longest Common Subsequence of 2 sets
  - Imagine 2 words, or 2 strings of DNA

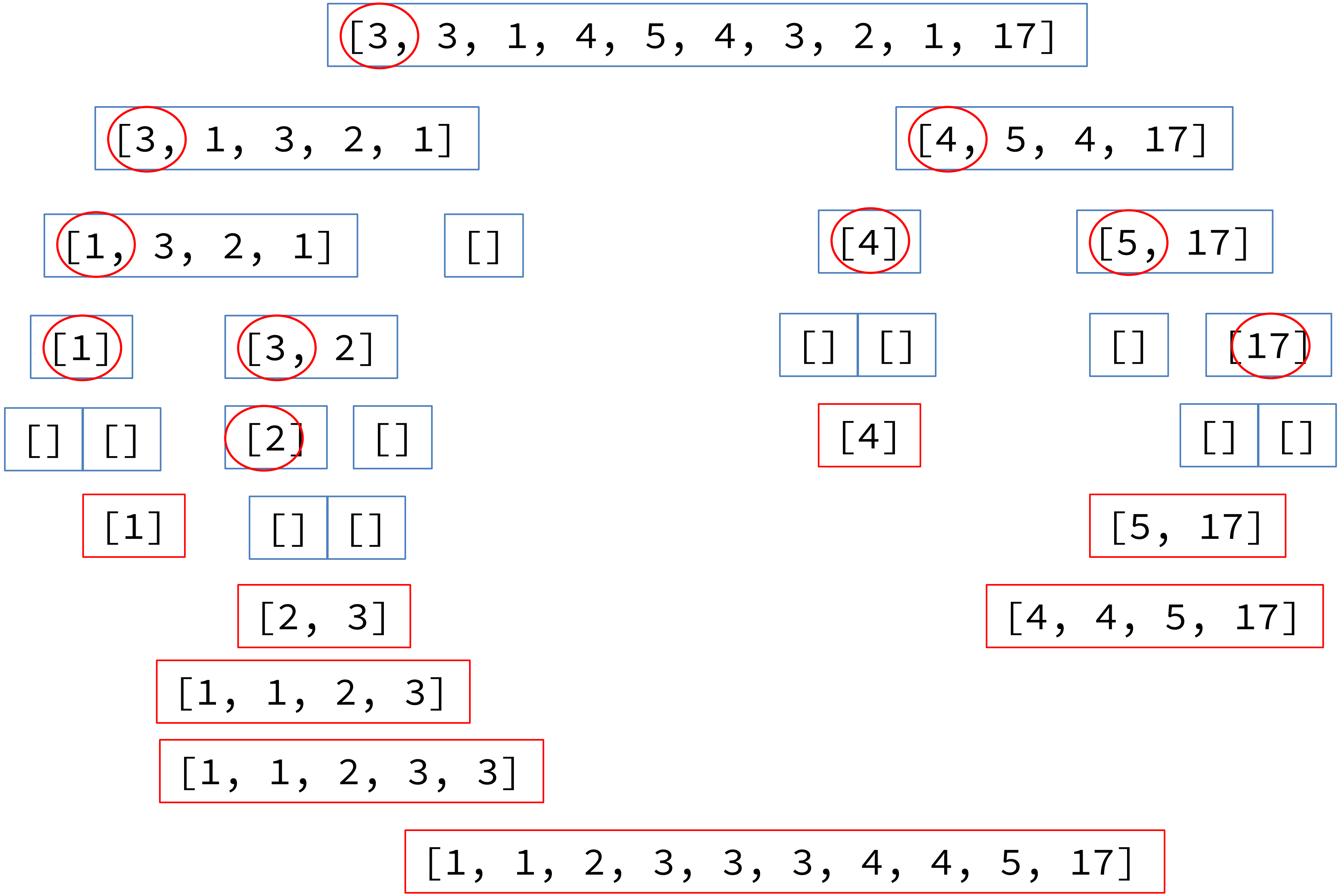
Computational Structures in Data Science

Bonus:
Quicksort

Quicksort

- A fairly simple to sorting algorithm
- Goal: Sort the list by breaking it into partially sorted parts
  - Pick a “pivot”, a starting item to split the list
  - Remove the pivot from your list
  - Split the list into 2 parts, a smaller part and a bigger part
  - Then recursively sort the smaller and bigger parts
  - Combine everything together: the smaller list, the pivot, then the bigger list

QuickSort Example



Tree Recursion

- Break the problem into multiple smaller sub-problems, and Solve them recursively

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def quicksort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = s[0]
        smaller, bigger = split(pivot, s[1:])
        return quicksort(smaller) + [pivot] + quicksort(bigger)

>>> quicksort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

Quicksort Visualization

- Interactive View